The Ising model as a discovery tool in cognitive sciences

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For decades, physicists have pursued the use ideas from statistical mechanics to capture the collective phenomena of life. Biological systems have a subtle structure that is not described neither by ordered crystals nor disordered gases. Moreover, these states are far-from-equilibrium, being maintained by a constant flow of energy and matter through the system. There are special states for a functional living system and, at the same time, their activity cannot rely on fine-tuning the parameters of the system. Of the many ideas originated in statistical physics that have been suggested to characterize these states, perhaps the most suggestive and speculative is the idea of self-organized criticality. The theory of self-organized criticality originated in models of inanimate objects (sand mountains, earthquakes, etc.) [1], [2], but then the theory was to include biological systems through the analysis of simple toy models [3]. A simple model the evolution of interacting species can self-organize into a critical state where the quiescent period is interrupted by avalanches of all sizes [4], describing a behavior similar to the idea of punctuated equilibrium in evolution [5]. Similarly, the brain has been suggested to be in a self-organized critical state at the boundary between being nearly dead and becoming completely epileptic [6].

Currently, some of the early ideas don't seem correct (for example, real sand-piles behave very differently from the model). However, the idea of biological systems operating near a critical point remains interesting. Over the last decade or so, there have been significant advances in experimental research on biological networks, suggesting a very different route to the use of ideas from statistical physics. Monitoring the activity or state of individual elements in a network has long been the traditional method, but now it is possible to monitor many elements in parallel. Technology is unique to each class of system. Large arrays of electrodes recording in parallel from multiple neurons [7], [8], high-throughput sequences probing large populations of amino acid sequences [9], accurate imaging to track individual animals in large groups [10], [11] — And of course each measurement has its own limitations. Nonetheless, as these new experiments became available, several groups tried to build statistical physics models directly from the data. These notable features Analysis scattered at many levels of the organization is the emergence of signatures of importance. Twenty-five years ago there was a grand theory that had little relevance to the data, but now there are many isolated arguments about specific experiments that suggest similar conclusions. Our goal here is to put together these analyzes. Perhaps it rekindles expectations for a more general theory.

The Ising model is a mathematical model of physical ferromagnetic material, used to study phase transitions, such as the transition from a ferromagnetic to a paramagnetic state in a material [12]. It can also be used to study the ordering of spins in a material, and to model spin glasses. The Ising model has been used to study the coordination of neural activity in the brain and to model learning and memory retrieval in neural networks [13]. In this contribution we will explore how the Ising model can connect different physical phenomena with the cognitive sciences.

Self-organized criticality (SOC) is a concept in physics and systems theory where a system spontaneously evolves towards a critical state. In a critical state, the system is "at the edge of chaos", meaning that it is highly sensitive to small perturbations and can exhibit large-scale fluctuations [1], [2]. SOC is thought to be a mechanism by which complex systems can self-organize and evolve. Many natural systems are thought to exhibit SOC, including earthquakes, solar flares, and forest fires. The Ising model has been a canonical example to study criticality in biological and neural systems. Using inverse learning methods to model neural activity using the Ising model, it has been suggested that the brain operates near a critical point, and that this could be a mechanism for the brain to self organize and evolve [14]. These methods have been later extended to broader classes of biological systems [15]–[17] and larger neural systems [18]. In addition, simple algorithms have shown how robotic agents can be driven to be near critical points spontaneously generating a diversity of complex behaviours, suggesting that criticality might be a simple mechanism to generate a diverse range of interactions with the environment [19].

Integrated information theory is a theory of consciousness developed by Giulio Tononi [20]. The theory is based on the idea that consciousness arises from the integration of information. The theory posits that consciousness is a measure of the amount of information that is integrated across a system [21], [22]. The more information that is integrated, the more conscious the system is. The theory has been used to explain a variety of phenomena, including the binding problem, the unity of consciousness, and the relationship between consciousness and the brain. Generally, the theory is difficult to explore in large systems due to its computational complexity. However, studies using the Ising model [23], [24] show that integrated information is maximized at critical phase transitions, suggesting a connection between self-organized criticality and integrated information in the brain.

Nonequilibrium thermodynamics is a branch of thermodynamics that deals with systems that are not in equilibrium. The field is concerned with the behaviour of systems that are far from equilibrium, such as those that are undergoing a chemical reaction or are subject to a strong external force. Nonequilibrium thermodynamics is a relatively new field, and it is still being developed. Some of the key concepts in the field include entropy production, irreversibility, and dissipative structures [25]. Nonequilibrium thermodynamics has been used to study a variety of systems, including chemical reactions, biological systems, and economic systems. Specially, it has been shown that irreversibility is a signature of the level of conscious activity in cognitive systems [26]. While the classical Ising model is a model of equilibrium thermodynamics, extensions of the model include irreversible dynamics with asymmetric connections [27]. Recent research shows that the Ising model can maximize its irreversibility when it is near a critical point [28], [29], suggesting a connection between nonequilibrium properties of neural networks and the capacity to self organize near critical points.

In conclusion, the Ising model shows the potential to connect different properties of cognitive systems with concepts from physics, exemplifying how new mathematical and statistical descriptions could be useful in capturing mental properties that are usually hard to trace analytically.

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