

Boyd, Pete ORCID: <https://orcid.org/0000-0002-2234-3595> and Ash, Andy (2018) Teachers framing exploratory learning within a text-book based Singapore maths mastery approach. *Teacher Education Advancement Network Journal*, 10 (1). pp. 62-73.

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**Teachers framing exploratory learning within
a text-book based Singapore Maths mastery
approach**

Teacher Education Advancement
Network Journal
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University of Cumbria
Vol 10(1) pages 62-73

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Abstract

South Asian approaches to teaching mathematics have been influential globally and are entangled with mastery approaches. This study investigates changing strategies and beliefs of Primary School teachers in England as they engage with a sustained curriculum development project. The teachers were working with a commercially available text book based scheme entitled Maths – No Problem!TM which is informed by Singapore Maths mastery approaches. The collaborative research involved seven teacher researchers in design, data collection and analysis. Data sources included classroom video and stimulated recall interviews with each of the seven teachers, plus a focus group in which the same teachers considered the role of the text books and two focus groups in which they contributed to collaborative data analysis. In lesson planning, supported by the scheme's text books and teacher guidance, the teachers appear to focus on maths subject knowledge as they anticipate how children might take different directions in exploring the 'anchor problem'. In classroom teaching, the teachers moved away from the use of in-class grouping and used questioning during a collaborative dialogic exploration phase of each lesson, before drawing together student solutions, supporting student journaling and introducing the text book to support reflective learning and practice.

Keywords

Mastery; Mathematics; Multidimensionality; Mindset; In-class grouping.

Introduction

In this paper we report on a practitioner research project with seven teacher researchers in Primary schools collaborating with the director of their school alliance and a university based research mentor. Teachers across the alliance of schools are engaged in a sustained curriculum development project focused on a 'mastery' approach to maths (Block & Anderson, 1975; Bloom, 1982; Cockroft, 1982). The schools are implementing a commercial scheme entitled Maths – No Problem!TM which provides textbooks, pupil work books and online teacher guidance materials. This scheme reflects a 'Singapore Maths' mastery approach; it is based on an officially approved Singapore text book scheme but amended to address the National Curriculum for mathematics in England. The curriculum development project is in its third year and displays many of the characteristics of effective professional development for teachers, including sustained involvement, engagement with external knowledge, classroom experimentation and evaluation and commitment to the project from school leaders (Higgins et al., 2015). This study asks the question: How does involvement in this sustained text-book based curriculum development project influence teachers' classroom strategies and beliefs about maths? In this paper we focus mainly on changes in the teachers' knowledge and beliefs around classroom strategies, including the role of the commercial scheme's text books within that. The study makes a small contribution to the large body of work on teacher strategies and beliefs in mathematics (Stipek et al., 2001; Cross 2009) and on differences between the approaches of South Asian and western teachers (Correa et al., 2008; Jerrim, 2014).

To contextualise the study, it is worth noting three characteristics of Primary school teaching in England. First, most Primary teachers in England are class teachers, that is they are allocated to an age

Citation

Boyd, P., Ash, A. (2018) 'Teachers framing exploratory learning within a text-book based Singapore Maths mastery approach', *TEAN Journal*, 10(1), pp. 62-73.

based class of around 30 pupils and teach them all week and for all or most curriculum subjects, including mathematics. Second, there is an embedded cultural and professional commitment to 'tracking' in England, meaning the use of setting or in-class grouping of children based on their prior attainment, despite research evidence that suggests few advantages and considerable problems with this strategy (Boaler, Wiliam & Brown, 2000; Kutnick et al., 2005; Francis et al 2016). Third, particularly since the late 1980s, there has been a policy focus on 'back to basics' including maths as a priority school subject with government interference in defining the curriculum and influencing approaches to mathematics teaching. This policy drive has been shaped by development of a high accountability regime for schools and by the modest performance of England in international mathematics test league tables when compared to high scoring Singapore and other East Asian countries (Ball, 2013; Mullis, Martin and Foy, 2016).

An additional contextual note relates to wider social and cultural beliefs about maths in the USA, in England and elsewhere (Tobias, 1995). Three assumptions are commonly made: that performance in maths is mainly about speed, solutions and calculation; that Maths 'ability' is inherited; and that being good at maths is a good measure of general intelligence (Boaler, 2016). These cultural beliefs are widespread and, to the surprise of international visitors, it is often deemed socially acceptable, perhaps even for a parent or teacher, to say 'I am rubbish at Maths'.

The Singapore Approach

The 'Singapore' approach to teaching maths has been interpreted and developed in a variety of ways within England but it aligns with key characteristics of 'mastery' approaches. Mastery itself is a contested concept but there are some shared characteristics between different interpretations of mastery approaches in maths. For example, there is an emphasis on whole class teaching and the children move at broadly the same pace with an emphasis on problem-solving (NCETM, 2016). Singapore Maths approaches tend to avoid in-class grouping and differentiation by task, both of which have been in widespread use in England (Micklewright et al., 2014). In terms of educational research there is an evidence base for the modest positive impact of 'mastery' approaches that rely on the social dimension, so the whole group or class must achieve an acceptable level of competence before anyone moves on from the topic being studied (EEF, 2015). Such a mastery approach to maths has influenced the National Curriculum in England and national guidance on mastery approaches to maths emphasises '...whole-class interactive teaching, where the focus is on all pupils working together on the same lesson content at the same time' and '...all can master concepts before moving to the next part of the curriculum sequence...' (NCETM, 2016). Central to this concept of mastery is the belief that, with appropriate conditions for learning, almost all pupils are capable of progressing in their knowledge and understanding (Block & Anderson, 1975; Bloom, 1982) and this creates a link to mindset theory (Dweck, 1999; 2006). This approach also points to a change in understanding of the nature of maths, of its multidimensionality, meaning that maths itself, and being a mathematician, are about much more than speed and calculation and include a wide range of problem-solving skills (Boaler, 2016).

Research evidence highlights wide variation in the success of mastery approaches, implying that teaching to mastery *effectively* is challenging (EEF, 2015). The approach discussed in this paper, uses textbooks as a teaching tool to provide an element of scaffolding for teachers attempting to apply the concept of mastery to their mathematics classroom. It has been suggested that dependence on textbooks could lead to the delivery of mathematics in a *deductivist* style, meaning maths is seen as a set of truths and is best taught through teacher demonstration followed by practice and high stakes testing (Hudson, Henderson & Hudson, 2015). However, it is argued elsewhere that well designed textbooks may be used appropriately as a supportive tool for mastery teaching (Oates, 2014).

The Singapore approach is centred on problem-solving and the classroom environment is developed to facilitate collaborative talk, referred to as ‘cooperative learning’ in Singapore, which aligns to heterogeneous grouping in which peer support can be utilised effectively (Lui, Toh & Chung, 2009). Table 1 provides an outline of a typical lesson within the curriculum development project.

Table 1. The lesson outline used by teachers in the project.

Lesson phase		Outline
Anchor Task	Exploring	One problem or stimulus is presented to pupils (based on what is in the textbook) and they are encouraged to explore it. The teacher uses this time to observe their responses and prompt further exploration with questioning to ensure that all pupils are challenged.
	Structuring	The teacher gathers together pupils’ ideas for solutions and the class discuss them as a whole group, often re-exploring new suggestions.
	Journaling	Pupils record what they have been doing in their maths journals – there is an emphasis on showing things in different ways and effective communication of thinking.
Reflect and refine		The textbook is used and the teacher guides the class through the textbook solutions to the problem they have been discussing. There is a greater emphasis on teacher explanation during this phase.
Practice		The teacher starts off by guiding the class through examples of similar problems to the one they have just done. Then, pupils work through more examples independently with the teacher supporting them if necessary. All questions are typified by their mathematical variation – they are designed to extend pupil’s thinking rather than just consist of many examples presented in the same kind of way.

The Singapore Mathematics approach also includes an emphasis on the Concrete-Pictorial-Abstract (CPA) heuristic (Yew Hoong, Weng Kin & Lu Pien, 2015) informed by Bruner’s *enactive*, *iconic* and *symbolic* modes of representation (Bruner, 1966). In exploring possible solutions to a problem, pupils move fluidly between using concrete manipulatives, pictorial representations and abstract symbols. Also, similar to Japanese lesson structures (Shimizu, 2002), each lesson follows a sequence (Table 1) that leads to the sharing and evaluation of their different solutions to a core problem, which is close to the Japanese idea of ‘neriage’ (Takahashi, 2008). In this way the lesson structure reflects a social constructivist approach where learning is initially pupil-led and the teacher provides scaffolding of ideas in order to help pupils acquire and refine new mathematical concepts (Hardman, 2008). As indicated in Table 1, another fundamental characteristic of this approach is the use of journaling. As an instructional tool in mathematics, journaling has been well researched and many positive outcomes found in a number of countries (Mason & McFeetors, 2002; Tan & Garcés-Bacsal, 2013).

The Study

This study adopted a collaborative practitioner research approach. A university-based researcher worked with a team of seven teacher researchers alongside the director of the school alliance as co-researcher over a two-year period. The research adopted a qualitative multiple case study design (Yin, 2014). The seven teachers worked in seven different schools and had all been involved in the school alliance Singapore Maths curriculum development project for between one and two years. Six of the teacher researchers had classes of children aged between 5 and 8 years old and one teacher researcher had children aged 10 and 11 years. The teachers had between 2 and 25 years of professional experience, six of them were female. The research team worked together on design, data

collection and collaborative analysis of data with an intention to create 'strongly contextualised' knowledge (Nowotny, Scott & Gibbons 2001; 2003).

For each teacher researcher a teaching assistant captured a maths lesson on video including a focus on one pair of children working together within that lesson and a short follow-up interview by the teacher with the two children. Semi-structured interviews with each teacher researcher used a form of video stimulated recall in order to help keep the teachers' reflections grounded in practice (Sherin & Han 2004; Schmid, 2011). The teacher was able to slow down or pause the video of their lesson at points where they wished to comment or in response to prompt questions from the interviewer (Lyle, 2003). This paper reports primarily on thematic analysis of transcriptions of the seven stimulated recall interviews (Braun and Clarke 2006) but includes two further sources of data. The significance of the text books emerged during interviews and a focus group with the 7 teacher researchers was used to generate additional data, with each teacher bringing along an annotated copy of text book pages related to a lesson they had recently taught. A third source of data consisted of transcribed audio recordings of two sessions in which teacher researchers contributed to collaborative analysis.

The thematic analysis involved reading and re-reading three of the interview transcripts of three interviews and initial coding using a constant comparative approach to develop an emerging index (Braun & Clarke, 2006). The developing coding was applied to the remaining transcripts and refined further. The continued analysis involved identifying and refining themes that 'capture' important elements within the data and the relationships between themes (Nowell et al., 2017). Teacher researchers contributed to a further session to refine the emerging analysis.

A key ethical risk related to the professional reputation of the teacher researchers who were sharing a lesson video for analysis. Teachers gave formal written consent and had a right to withdraw. Transcript data was anonymised and pseudonyms are used. Children gave oral consent and parents and carers gave written consent and had a right to withdraw. The project proposal was subjected to a formal university ethical clearance process.

Findings: Framing Learning in Maths

This section presents three themes, generated from the collaborative data analysis, of dialogue, exploring and concept-building. These themes are discussed with illustrative quotations from the teacher researchers.

Dialogue

A focus on verbal reasoning, on classroom dialogue focused on problems in Maths, is a dominant theme emerging from analysis of the four teacher interview transcripts:

I would say one of the impacts of the whole approach in general is that when the children are talking in lessons, their mathematical vocabulary, their reasoning, it's overtly obvious to you the change from the way things were previously...

(Andrew).

Here the teacher is referring to the classroom video and pointing out their perception of a significant shift in their practice. The teachers emphasise the development of confidence and skills by themselves and by their children in talking about Maths problems. On observing classroom video of children discussing Maths problems the teachers refer to explicit training and development of dialogic skills:

...since we've been doing the Singapore Maths, obviously because it uses a lot of talk partners, I've trained them...in how to speak to each other so in the video...you hear a lot of phrases that

sound a little bit scripted, because...I've taught them to use certain words when they're talking to each other...

(Patricia).

The teachers point out that this journey of developing classroom dialogue has not been easy for all of their children:

We've also got some children who have struggled...it's been quite a real journey getting them to communicate...their level of communication, that's been something we've had to really focus on; talking to each other

(Rachel).

But overall the teachers show a commitment to developing dialogue around maths problems in their classrooms. They insist that their children are developing confidence and verbal reasoning:

The little girl...she stood up and was saying 'she must have just looked away and 3 balloons had gone'...she's actually not a very confident mathematician, but she's confident enough in front of everybody...whereas I think maybe in the past that wouldn't have been the case...

(Audrey).

The emphasis on dialogue indicates a change in the classroom learning environment that the teachers are creating in their Maths lessons. The teachers use slightly varied approaches to putting the children into pairs during the lessons: One teacher uses random selection of 'talking partners' and changes them regularly; four teachers put the children in pairs selected deliberately to have different prior attainment in maths; and two teachers pair the children based on 'how well they cooperate together'. Three of the teachers also use 'zoning', so that there are clusters of pupils in particular areas of the classroom where one child in each pair has special learning needs or is considered to have a particularly high level of prior attainment.

At the start of lessons, the teacher introduces the anchor problem, but the emphasis is on presenting a challenge rather than making clear explanations of neat solutions. Consolidation is achieved in a plenary activity through dialogue:

...the shift from the old style didactic teaching with the teacher stood at the front saying, 'here is the rule of the day'. The children are gaining ownership of emerging rules, the mathematical rules are coming almost through the children's exploration, they are not explicitly being taught and that's something very much that I notice in my classroom, those rules, those conventions of maths, they appear through discussion rather than being explicitly taught...

(teacher researcher during collaborative analysis).

This comment is illustrative of a pedagogic change that is closely entangled with a change in understanding of the nature of school Maths as a subject. The teachers are aware of the change that the mastery approach is making to children's capacity for verbal reasoning:

I went into Year 6 [10 or 11 years old] where they never did any Singapore Maths. I was taken aback really about how difficult they found it to express their ideas, and then went into a Year 3 [6 or 7 years old] today who've had two years of Singapore maths and they knock spots off the Year 6...

(teacher researcher during collaborative analysis).

Maths is seen by the teachers as more about thinking in depth around problems rather than quickly

solving algorithms. The importance of dialogue, rather than teacher monologue, is emphasised by the teachers:

I think one of the main differences is that we learn to shut up...so we wouldn't tell them what to think straightaway, it's kind of, 'well what do you think?' and they have to go through their own journey of unpicking the maths and exploring ideas and then once they've got this pool of ideas it's almost like we're using that pool to direct and scaffold the lesson
(teacher researcher during collaborative analysis).

This quotation includes the term 'exploring' and this is a key difference identified by the teachers between their previous approach to Maths their current practice developed through their engagement with the mastery approach project. Overall, the dialogue theme captures the change in classroom strategies but more than that, indicates a shift in the perspectives of the teacher researchers about the nature of 'school maths'.

Exploring

This emphasis on verbal reasoning, on classroom dialogue, reflects a view expressed by the teachers that Maths is about exploring problems:

...throughout the year they realised that there's going to be something for them to work out...and they have to share it with each other and talk about it and explain a reason why... Whereas I feel like in the past...they have just thought 'oh that is something to do with maths, I'll just wait until Miss tells me what we're doing'
(Lisa).

In this illustrative quotation Lisa signals some of the perceived changes that teachers identify in their new mastery approach to Maths compared to their previous practice and which we conflate here as 'exploring'. The shift is complex and includes overlapping elements including changes in dialogue, collaborative learning, teacher role, and underlying beliefs about Maths and about intelligence.

As part of framing exploration activity the teacher researchers place a high value on the text books in the commercial Singapore Maths scheme that their project uses. They refer to the 'anchor problems' offered for each lesson by the text. The teachers refer to their use of low threshold tasks, meaning accessible tasks that are challenging to explore but that all children are able to engage with initially at a basic level:

...you've got something [an anchor problem] that on the surface looks like - well if every child in your class can't answer that you've got problems my friend...so you go 'oh you can do that so let's do something else' and it's about this deepening of understanding...the deepening understanding the children have got and yeah, the way that they reason and the way that they think...
(Andrew).

This focus on an anchor problem also affects an element of pace, that more time is spent exploring one problem rather than moving on to consider other tasks:

...because they're talking to each other so much and they're going deeper and deeper into the problems...one of the ways Singapore Maths has changed the way I teach is that you spend a lot of time on one problem, rather than jumping from problem to problem so you do get that deeper understanding and you don't really leave anybody behind...
(Patricia).

The teachers are not differentiating by task but claim to use facilitation, especially questioning, to maintain a high level of challenge:

...in this bit...I'm more facilitator...when they're actually discussing it with each other I almost become unimportant in the room...if I can see that they've got a good idea; they're almost there so I'm sort of asking a question 'well just tell me about why you've done this little bit here'. Can you tell me a little bit more?' It's just sowing a little question to maybe push them on...

(Rachel).

When prompted to discuss changes in their teaching of Maths the teachers highlight their use of questioning during the time that the children spend on exploring the anchor problem:

I suppose it's not questioning what answer the children have got but *how* they got there. A complete shift in what you want to know so some of the children might say - the problem might be about 40 apples and they say 'I know that's 40 apples' and they know that I want to know how they know that...

(Veronica).

This reliance on teacher questioning is not seen as dominating the classroom even though the teacher is being very active, moving around the room and using questioning to prompt further exploration:

It's not teacher-led now. As I say we're there to facilitate. It's now about the children taking control of their own learning and taking ownership, particularly in their journals, taking ownership of their thinking and their knowledge. Extending their own knowledge and understanding as well...

(Kirsty).

This quotation illustrates the teachers' claim that, in association with their use of questioning, the children are learning more independently.

Although the teachers referred to their role within the exploring activity as 'facilitation', we preferred to use the metaphor of 'framing'. This is because the children seem free to explore the anchor problem but actually the teacher is proactively orchestrating the learning, largely through questioning:

...it's kind of structured by the teacher I think... and you're making key decisions in the lesson...which avenues to explore, because there are so many of them...and it's kind of 'oh well, do we want to pursue that one at this point?'. So, you're still making decisions then, to guide the lesson...

(teacher researcher during collaborative analysis).

This comment illustrates the way that the anchor problem from the textbook is clearly structuring the lessons but the teachers are aware of the way that their subject knowledge is critical for their framing of the exploring activity. Although the anchor problems are designed to be low threshold, meaning they are accessible for all of the children, the teachers note that for older children the problems are not so straightforward even for the teacher:

...when you first start using this [*maths scheme*] textbook, for some people it can knock confidence a little bit as well. I'm thinking further up the school, Year 5 [aged 9 and 10 years old]. If you're looking at a problem and you're not understanding it and you're thinking, 'how did I teach this before?', and it's knocking confidence a little bit as well as improving it

(teacher researcher during collaborative analysis).

The role of the text books was strongly foregrounded by the teachers but they emphasise the need to prepare lessons carefully and to engage with the teacher guidance and training:

...that's the thing with student [teachers] especially, if they've not had the training and you just try and pick up a textbook and teach a lesson, probably if you did pre-Singapore Maths style with the [*maths scheme*] textbook, it can turn into a really bad lesson, very dull. You have to have the training and the principles with the textbook otherwise it doesn't work at all...

(teacher researcher during collaborative analysis).

The significance of the text books and additional teacher guidance became very clear during the initial teacher interview analysis and this led us to hold a teacher focus group specifically engaging with this issue. Each teacher researcher brought a copy of the text book pages for a recent lesson and discussion built from these specific examples to general comments about the significance of the text books:

I agree that the textbooks and workbooks have freed up my time from producing worksheets to being able to think about mathematical learning and to really unpick how children learn and how I can scaffold

(Audrey).

It's revolutionised my teaching. My subject knowledge is beyond anything it ever was. I enjoy maths, I have an enthusiasm for maths and I think the depth of rehearsal I go through for my lessons, I would never, ever have had that freedom or time to do it if I didn't have the textbooks

(Rachel).

These quotations illustrate the views of the teachers about the significance of the text books and their impact on development of subject knowledge in Maths. The structure provided by the textbooks seems to free teachers from more mundane aspects of lesson planning and teachers' preparation for the exploring phase of the lessons appears to demand development of subject knowledge. Overall, the appropriate use of these text books does not appear to lead towards a deductivist style of maths teaching (Hudson, Henderson & Hudson, 2015).

Concept-Building

The teachers appear to be using listening and questioning to make formative assessments of children's progress. They claim to spend considerable time on exploring problems and see this as developing 'deep understanding' of key concepts within Maths:

...so the difference in teaching...is the amount of time spent really digging deeper into cognitively what we're thinking. Conceptually what is it that we're talking about and giving the children that conceptual understanding of it...they are then able to apply that understanding as they move forward...it will have a lasting effect on them...

(Andrew).

The teachers are in their first or second year of adopting the new approach to Maths and are aware of differences in their expectations around concept-building:

...so this is the beginning of my second year of teaching Singapore Maths and it's completely changed my way of teaching and I think probably not just in maths actually. I would say it's making it more open and...the children now expect to be asked to think at a deeper level and to explain their thinking...

(Audrey).

This shift towards in-depth understanding is a key claim made by all of the teachers and reflects the emphasis of the Singapore Maths approach on an interpretation of teaching for 'mastery':

...there were children at the end of that lesson that couldn't add 30 minutes and an hour, but I could have just said to them 'right if it's on the 6 it's half an hour'...and then they would have been able to answer questions in a book but they wouldn't understand the concept. This is a lot more about children understanding concepts...

(Patricia).

In this quotation the teacher illustrates the belief in a long-term investment in deep understanding which all of the teachers seem to feel is a requirement of adopting the Singapore Maths approach. In the context of the high accountability educational policy framework in England this represents a brave stance.

Mindset, Grouping and School Maths

The study generated a rich body of qualitative data and our analysis has gone beyond the changing classroom strategies and influence of the text books to consider changes in wider underpinning beliefs of the teacher researchers. In a second paper we will report in detail on how engagement in mastery approaches appears to be changing teacher beliefs around the three inter-related areas of: maths mindset (moving towards high expectations of all children in maths); grouping of children by prior attainment (that this is no longer relevant within a mastery maths classroom); and the nature of school maths (its multidimensionality and a changed understanding of what it is to be a mathematician).

Discussion

The teachers are facilitating collaborative learning supported by visualisation and questioning to provoke in-depth exploration of the anchor problem and relevant key concepts through verbal reasoning. This mastery approach to positioning problem-solving at the heart of school maths is aligned to the recommendations of the Cockcroft report (1982) mediated via Singapore Maths and now explicit in the National Curriculum in England (DfE, 2014). Implementing this strategy includes changes in the learning environment to embrace struggle and mistakes and for the teacher to model verbal reasoning to slowly explore problems in depth rather than demonstrating quick neat calculation to reach the 'correct' answer to questions. We characterise this approach as 'framing' learning and a distinctive feature is that the teachers are moving away from grouping by prior attainment (Francis et al., 2016). Our research question asked about change in classroom strategies and within the constraints of our small-scale study teachers implementing this commercial Singapore Maths scheme do appear to have moved strongly away from a deductivist style of teacher demonstration followed by practice and high stakes testing (Hudson, Henderson & Hudson, 2015).

The teachers are involved in a multi-school curriculum development project with considerable support including workshops, online guidance and classroom observation. The project includes the features of effective professional development for teachers (Higgins et al., 2015) and it is not surprising that their teaching strategies have changed. What the study reveals is the ways in which the teachers have also changed their beliefs about the importance of dialogue, the value of collaborative exploration, the impact of in-depth concept-building and these changed beliefs point towards a view of maths as multidimensional (Boaler, 2016). Schools making a decision to invest in such a mastery maths scheme will need to accept the accompanying challenge of changing teachers' beliefs. Our small study did not necessarily include teachers who are resistant to change and changing beliefs is intertwined with changing strategies but presents a higher hurdle.

Our research question also refers to the text books. The teachers rely heavily on the text book Singapore Maths scheme to provide a carefully selected anchor problem, a well-constructed practice tasks that include variation, and relevant teacher guidance. The teachers find that their preparation is focused on mathematics subject knowledge, because the children might take the anchor problem in different directions. The teachers value the way the mastery scheme gives them ‘permission’ to spend classroom time on exploration of the anchor problem. The approach to teaching mathematics has considerable overlap with other attempts to move away from grouping by prior attainment in order to address social justice issues (Boaler, 2008; Boaler 2016). The teachers initially found dependence on a text book based approach disconcerting, but consider that they have grown to trust the scheme as they gain familiarity with the sequences of lessons. The dependence on the textbooks and associated teacher guidance may create a concern for schools, because it might be part of reducing teachers to a technician role of ‘delivering’ the curriculum. It also creates a considerable pressure to be confident that the textbooks and guidance being invested in are evidence-based and effective (Oates 2014). Despite the considerable support teachers feel from the text book scheme and its teacher guidance, this mastery approach appears to demand skilful and committed teachers in particular to frame the whole class exploratory learning. Schools implementing such a scheme face the risk that some teachers may not fully engage and that teachers with weak subject knowledge may struggle.

The limitations of this study mean that it is not able to provide any endorsement of the efficacy of the Maths – No Problem!TM text book based scheme except with respect to the perspectives of our small sample of teacher researchers. The study has highlighted possible areas for further research and development work related to our understanding of mastery approaches to mathematics teaching. This includes: the nature of ‘problems’ and ‘problem-solving’; the influence and embedded pedagogy of text books; the significance of teacher subject knowledge; the impact of teacher questioning during exploratory learning and how it relates to levels of challenge for all pupils. Overall, the study has indicated that developing mastery approaches in mathematics is a change project with both technical and cultural elements that challenges teachers to re-evaluate their classroom strategies but also their underpinning beliefs.

Acknowledgements

Seven teacher researchers contributed significantly to this research project, they are Lucy Evans, Ann Kirk, Rosie Ross, Paula Spenceley, Vicky Stout, Adam Vasco and Keri Williams.

Research Funding

This research project was funded by the Deep Learning Teaching Schools Alliance.

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