Teaching and Teacher Education 75 (2018) 214–223

Contents lists available at ScienceDirect

Teaching and Teacher Education

journal homepage: www.elsevier.com/locate/tate

Mastery mathematics: Changing teacher beliefs around in-class grouping and mindset

Pete Boyd [a, *], Andy Ash [b]

[a] University of Cumbria, United Kingdom
[b] Deep Learning Teaching School Alliance, United Kingdom

HIGHLIGHTS

- A Singapore Maths mastery approach influences teacher strategies and beliefs.
- The text-book based scheme shifts teacher planning towards subject knowledge.
- Teachers' relinquish their commitment to grouping by prior attainment.
- Teachers' cultural beliefs about the nature of mathematics are changed.
- Teachers comments suggest beliefs in malleable intelligence in mathematics.

ARTICLE INFO

Article history:
Received 31 July 2017
Received in revised form
20 June 2018
Accepted 22 June 2018

ABSTRACT

Internationally, reform efforts in teaching of mathematics have found it difficult to change practice. This study used classroom video stimulated recall interviews with Primary teachers in England to investigate their beliefs during implementation of a textbook-based South Asian mastery approach to teaching mathematics. The self-reported beliefs of the teachers showed their support for change in practice, from in-class grouping by prior attainment to whole class teaching with everyone exploring the same problem, and suggested conceptions of malleable intelligence in the specific domain of school maths.

1. Introduction

The beliefs of teachers have been identified for some time as a challenge for the reform of mathematics instruction (Stipek, Givvin, Salmon, & MacGyvers, 2001). These beliefs are partly subject specific, related to the nature of school maths (Correa, Perry, Sims, Miller, & Fang, 2008). This study contributes to the international body of work by focusing on a small group of teacher researchers working in schools in England that are implementing a mastery maths text book based scheme informed by a Singapore Maths approach. The Singapore Maths approach was developed by the Singaporean Ministry of Education during the 1980s in a scheme that depends on use of text books and includes aspects of a mastery approach, for example by spending more time to investigate each topic in depth. In our study we do not seek to associate raised pupil attainment with a mastery approach, rather we investigate the complexity of change in classroom practice and of teacher beliefs within the context of adopting such an approach to maths. The commercial scheme, including textbooks, pupil work books and teacher guidance materials, is based on a scheme used in Singapore but has been amended for use in England and is entitled *Maths - No Problem!*™.

Previous research has investigated mathematical subject knowledge and beginning teachers' beliefs (Cooney, 1985; Paolucci, 2015). In this study the focus is on experienced teachers' underpinning beliefs during implementation of a practical mathematics scheme engaging them in new pedagogical knowledge as well as new mathematics subject knowledge. The seven teacher researchers contributing to this study were in years two and three of a wider and sustained curriculum development project that included three initial workshops, classroom experimentation, supportive classroom observation with coaching on two or three occasions, and primarily provision of the textbooks, workbooks and teacher guides that form the Maths - No Problem!™ scheme. Teachers may have different conceptions of mathematics as a discipline and of 'school maths'. It seems likely that their personal experience of

* Corresponding author.
E-mail address: pete.boyd@cumbria.ac.uk (P. Boyd).

https://doi.org/10.1016/j.tate.2018.06.016

schooling, as a pupil and then as a teacher, will be a strong influence alongside any formal higher education experience of mathematics they may have gained during an undergraduate degree or teacher education programme (Beswick, 2012). Teacher beliefs are a significant influence on their classroom practice and are relatively difficult to change despite the efforts of teacher educators and policy makers (Meirink, Verloop, & Bergen, 2009). It is important to consider individual teacher beliefs, for example in the way that might involve resistance to reform approaches to teaching, but also to consider the influence of teacher discourses and professional relationships on their beliefs (Priestley, Biesta, & Robinson, 2015). This study will consider beliefs to be on the same dimension as knowledge: 'Those things we "more than believe" we refer to as knowledge and those things we "just believe" we refer to as beliefs' (Leatham, 2006, p. 92). The study adopts a collaborative practitioner research approach to ask the question: What are the self-reported pedagogical beliefs of teachers during the implementation of a mastery maths curriculum development project?

2. School maths

In relation to this study, it is important to consider how mathematics as a subject discipline is transformed to become 'school maths' (Bernstein, 2000). Bernstein identified three areas of rules by which knowledge is transformed from its site of production, for example by researchers in the university, to the classroom, to become the content of lessons in schools. These three areas consist of: distributive rules, related to knowledge production; recontextualising rules, related to official curriculum policy and the local pedagogical influence of teachers; and evaluative rules, related to reproduction of knowledge by pupils in classrooms, tests and examinations (Bernstein, 2000; Puttick, 2015). With regards to mathematics, this process appears to be influenced by beliefs held about the contested nature of the subject itself.

Lakatos (1976) identifies two contrasting perspectives. On one hand, influenced by Euclidean methodology with its deductivist style, mathematics may be conceived as a set of eternal, immutable truths. This mathematics may be transformed (Bernstein, 2000) into a form of school mathematics that is associated with a pedagogy dominated by teacher demonstration followed by individual practice and high stakes testing with strict rules and right or wrong answers (Hudson, Henderson, & Hudson, 2015). On the other hand, mathematics may be seen as a human activity, and it is this human mathematical activity that produces mathematics – referred to as 'mathematics in the making' by Polya (1957, p. xxxii). This mathematics may be transformed into a school mathematics that presents the subject as 'fallible, refutable, and uncertain and which promotes critical thinking, creative reasoning, the generation of multiple solutions and of learning from errors and mistakes' (Hudson et al., 2015, p. 377).

Within British and American culture this transformation of mathematics into a school subject can be seen to be significantly influenced by two inter-related and damaging myths concerning mathematics: first that only some people, due to natural talent, can do mathematics; and second that being good at mathematics is a clear sign that you are one of the most intelligent people (Boaler, 2016). The influence of these myths on children is revealed in studies of their disposition towards maths based on the language they use (Mazzocco, Hanich, & Noeder, 2012) and their level of anxiety around maths (Maloney, & Beilock, 2012; Ramirez, Chang, Maloney, Levine, & Beilock, 2016).

Perhaps as a result of these cultural myths, maths teaching in schools in Western cultures tends to consist of teacher demonstration and telling, followed by individual practice. Attempts to move towards a more constructivist, active learning approach in mathematics, one that might contribute somewhat to mathematics being more like Polya (1957, p. xxxii) 'mathematics in the making', have struggled to become embedded in schools (Hudson et al., 2015).

There are a number of important factors to consider as to why this may be the case. In Primary schools in England, for children aged 4–11 years old, the teachers generally teach a class across the curriculum, they are not mathematics specialists. Another significant contextual factor is that there is a high accountability policy framework in England (Ball, 2013) including a detailed National Curriculum with high stakes school inspections. The individual educational and workplace histories of teachers also influence their beliefs about maths and how to teach it as a school subject (Rogers, Cross, Gresalfi, Trauth-Nare, & Buck, 2011). Previous major reform influencing the teaching of mathematics in Primary schools in England has highlighted the significance of teacher identity and the need for professional learning to explicitly engage with this element of becoming a mathematics teacher (Brown & McNamara, 2011). In particular, as they develop their identity as teachers of mathematics during engagement in a curriculum development project, teachers are likely to change their criteria for successful learning in important ways, especially if they have been helped to focus on the development of children's mathematical thinking (Gabriele & Joram, 2007). Teachers may feel most effective when they teach mathematics by 'telling' and this is connected to the belief that the answers to mathematical problems are in books with the teacher acting as interpreter (Smith, 1996, p. 391). Alternatively, Cognitively Guided Instruction (CGI) emphasises teachers gaining an increased understanding of children's development of mathematical thinking during problem-solving (Fennema et al., 1996). Working with CGI, Moscardini (2014) argues for a view of pedagogy that includes underpinning values and beliefs and he highlights in particular values related to inclusion.

3. Singapore mathematics

Many schools in England are adopting mastery approaches to maths teaching influenced by South Asian approaches. The approaches vary somewhat but share some common characteristics. The whole class, of mixed prior attainment, moves at broadly the same pace through a maths curriculum that is not too content heavy. Lessons usually begin with whole class engagement with a contextualised problem. Collaborative work includes considerable use of concrete materials, manipulatives, and dialogue around possible solutions. The tasks are carefully selected and there is an emphasis on mathematical variation and connections. In line with early thinking on mastery learning there is an underpinning assumption that, under appropriate instructional conditions, virtually all students can and will learn most of what they are taught (Block & Anderson, 1975). It is important to note that Singapore Maths is strongly influenced by international theory and research evidence. For example, a key principle of the Singapore approach is known as the Concrete-Pictorial-Abstract heuristic (Yew Hoong et al., 2015) which is based on Bruner's *enactive, iconic* and *symbolic* modes of representation (Bruner, 1966). The Maths – No Problem!™ text book scheme is based on a scheme approved in Singapore and adapted for use in England. Compared to widespread practice in England the Singapore approach places less emphasis on differentiation by task or content and largely avoids in-class grouping by prior attainment (Micklewright et al., 2014). It is both a strength but also a limitation of this study, that it focuses only on teachers using Maths - No Problem!™

In a large-scale study of primary schools in England using a mastery approach influenced by Singapore Maths, a modest impact on learning was identified (EEF, 2015; Jerrim and Vignoles, 2016).

However, importing strategies from other cultures is not straightforward. A useful study investigated the maths achievement of western born children of East Asian descent living in Australia and showed that they performed more strongly in maths no matter what teaching strategies were used in their schools (Jerrim, 2015). Alongside this, Wong et al. (2009) suggest that, due to the unique cultural, political, religious and racial composition of Singapore, their approach to mathematics education is unable to be emulated in other countries. Despite this, it can be seen that many aspects of the approach are based upon seminal research and theory from around the world as well as being inspired by the British Cockroft report (Cockroft, 1982; Wong et al., 2009). In this study we investigate teachers' beliefs as they implement the Maths - No Problem!™ textbook scheme within a sustained curriculum development project. The textbook scheme includes student workbooks and web-based teacher guidance materials to accompany the textbooks.

In this study the teachers are generally using a lesson structure developed across the alliance of schools in combination with the use of the commercial Maths - No Problem!™ scheme. The general lesson structure is set out in Table 1 and includes exploring, structuring and journaling before the textbook is introduced. The Maths - No Problem!™ scheme, as developed by the alliance of schools, aligns with broadly agreed principles for mastery approaches (NCETM, 2016). However, the schools vary, for example in their approach to interventions with students who are considered not to have achieved the required level of mastery in a particular lesson or topic.

In understanding classroom practice in school maths in England it is important to take account of these strong and interwoven cultural, educational, social, commercial and even political influences as they help to shape the classroom practice of a teacher and their interactions with individual children.

4. Mindset and mathematics

In social-cognitive theory, an individual's sense of self-efficacy, their belief that they can have some control over aspects of their life to achieve particular performances, is a major influence on their personal agency (Bandura, 1986) and has an impact on motivation and academic performance (Schunk, 1991). Developing manageable goals and experiencing mastery by achieving them is motivating and helps to develop self-efficacy (Bandura, 1997). There has been a considerable focus on self-efficacy in maths showing that performance, for example in problem-solving, is strongly influenced by self-efficacy because the individual's beliefs affect the way they use their existing knowledge and skills (Michaelides, 2008; Pajares & Kranzler, 1995). Self-efficacy in maths is generally measured by asking research participants, using a Likert type scale, to judge their confidence in solving particular maths problems. There are key principles around specificity in measuring self-efficacy by relating it closely to particular tasks (Michaelides, 2008).

Related to the broader concept and body of work on self-efficacy, Carol Dweck and colleagues have focused on beliefs around fixed and malleable intelligence and developed mindset theory (Dweck, 1999; 2006). Mindset varies along a dimension characterised by fixed or growth mindset and is measured using responses to a Likert type scale focused on beliefs about intelligence (Dweck, 1999). A growth mindset is a belief that the harder you work, the smarter you get. It may be explained in an accessible way to learners by considering the brain to be a muscle, the more you exercise it, the stronger it becomes. The reason for explaining mindset to learners is that it is possible to nurture the development of a growth mindset (Dweck, 1999, 2006). One of the weaknesses of much mindset theory research is that even when it does focus on beliefs around intelligence within the context of schooling it is often generic, meaning it is across the curriculum. There is a need from more research on domain specific mindset, for example in particular curriculum subjects such as maths. Even Mindset Theory studies that are set within the specific domain of maths will often evaluate the impact of a generic Mindset training intervention, rather than the impact of a reform maths approach that may influence Mindset (Blackwell, Trzesniewski, & Dweck, 2007). In this study we are interested in the possibility of 'situated mindset', meaning teacher or pupil belief about intelligence within the context of school maths (Boaler, 2016) rather than a more general mindset (Dweck, 1999, 2006; Hymer & Gershon, 2014).

In England, projects such as 'Learning Without Limits' have demonstrated the need to work initially with teachers and other adults working in a school in order to challenge underlying beliefs and language before considering the impact of labelling children (Swan et al, 2012). Mindset is related to differences in cultural beliefs, for example in the way individuals in Japan and North America attribute reasons for failure and respond to it (Heine, Kitayama, Lehman, Takata, Ide, & Matsumoto, 2001). Mindset beliefs seem likely to be tangled up with children's experiences of social class and disadvantage and a study in Chile has shown how growth mindset beliefs may help to mitigate the effects of poverty on attainment (Claro, Paunesku, & Dweck, 2016).

A considerable challenge arises in considering self-efficacy and mindset, which is that the teacher's self-efficacy and mindset affects their classroom behaviours and has considerable influence on the beliefs of their students (Tschannen-Moran and Woolfolk Hoy, 2001). In the specific context of school maths, studies have shown how teacher beliefs influence their design and facilitation choices in the classroom (Cross, 2009). Levels of teacher enthusiasm and self-efficacy are related to classrooms in which children are more likely to adopt mastery-oriented goals (Lazarides, Buchholz, & Rubach, 2018) and these are the kinds of classrooms that are likely to nurture children's development of growth mindset. The

Table 1
The lesson outline used by teachers in the project.

Lesson phase		Activity
Anchor Task	Exploring	One problem or stimulus is presented to pupils for them to explore. This 'anchor' problem comes from the text book, but the books themselves are not yet introduced into the lesson. The teacher uses this time to observe pupil responses and prompt further exploration with questioning to ensure that all pupils are challenged.
	Structuring	The teacher gathers together pupil's ideas and the class discuss them as a whole group, often re-exploring new suggestions made.
	Journaling	Pupils record what they have been doing in their maths journals – there is an emphasis on showing things in different ways and effective communication of thinking.
Reflect and refine		The textbook is used and the teacher guides the class through the textbook solutions to the problem they have been discussing. There is a greater emphasis on teacher explanation during this phase.
Practice		The teacher starts off by guiding the class through examples of similar problems to the one they have just done. Then, pupils work through more examples independently with the teacher supporting them if necessary. All questions are typified by their mathematical variation – they are designed to extend pupil's thinking rather than just be lots of examples presented in the same kind of way.

significance of teacher beliefs is further complicated by the cultural dimension of self-efficacy and this is relevant to approaches such as Singapore Maths because studies have shown differences in the beliefs, for example, of Asian origin students compared to non-immigrants living in western countries (Jerrim, 2015; Klassen, 2004).

5. Grouping by prior attainment

Research review indicates that grouping students based on prior attainment involves a wide range of practices and that the impact of grouping is complex (Kutnick, Sebba, Blatchford, Galton, & Thorp, 2005). This research review highlighted that approaches to in-class grouping may need to vary depending on age and curricular area. In-class grouping by prior attainment, particularly in mathematics, has been widespread across Primary schools in England. A more recent review clarifies the position that overall, setting or streaming by prior attainment does not have a significant impact on levels of attainment except for a negative impact on lower sets and streams (EEF, 2015).

Unfortunately, this generally agreed position on the impact of grouping has not been taken on board by policy makers in England. This has arguably influenced how schools, school inspectors, teachers and even the general public, consider grouping by prior attainment (which in schools in England is generally and misleadingly referred to as grouping by 'ability'). This situation has implications for social justice and it is possible to identify at least seven key problems that may cause low attainment by students allocated to low sets or streams (Francis et al., 2017):

1. Misallocation to groups;
2. Lack of fluidity of groups;
3. Quality of teaching for different groups;
4. Teacher expectations of pupils;
5. Pedagogy, curriculum and assessment applied to different groups;
6. Pupil perception and experiences of grouping by prior attainment, and the impact on their learner identities;
7. These different factors working together to cause a self-fulfilling prophecy.

The current study involves all of these potential problems because in Primary school classrooms in England, for pupils aged 4–11 years, the classes are usually mixed in terms of prior attainment but the use of in-class grouping is widespread, particularly in maths lessons. Discourse analysis of government policy documents and associated debates in England helps to explain why it has proved very difficult to shift teacher beliefs in England around the effectiveness of setting, streaming and in-class grouping by prior attainment and the approach particularly appeals to middle class parents as part of a 'natural order' (Francis et al., 2017, p. 7).

6. Developing school maths differently

The work of Jo Boaler has highlighted the issue of Mindset Theory and how it relates to grouping based on pupils' prior attainment within the specific domain of mathematics (2016). For example, a large scale longitudinal study of 14 years olds using interviews, observation and test results identified some disadvantages for students in top sets as well as for those in bottom sets (Boaler, Wiliam, & Brown, 2000). Top set students were required to learn at a pace that was 'incompatible with understanding'.

In her recent text Boaler addresses the issue of how to effectively teach maths in heterogeneous (mixed) groups of children in such a way that students can be encouraged to take mathematics to different levels (2016, p. 115). The key strategy proposed is termed 'complex instruction' (Boaler, 2016, p. 118; Cohen & Lotan, 2014). Boaler argues that complex instruction contributes to tackling social justice issues arising for diverse students in school mathematics. Similar to Polya (1957, p. xxxii) 'mathematics in the making', this approach promotes an emphasis on 'multi-dimensionality' which teaches children that maths involves asking good questions, proposing ideas, connecting different methods, using calculation and evaluating the proposed solution (Boaler, 2008).

Bernstein's contextualisation rules for the knowledge power transformation of mathematics as a subject discipline to become school mathematics (2000) provides a useful theoretical framework for our study. Our purpose is to investigate change in teacher strategies and beliefs within the complexity of their classroom as they play their part in recontextualisation.

7. Methodology

This study is collaborative practitioner research involving a university-based researcher working with a team of seven teacher researchers based in seven schools, with the school-based director of the schools alliance as a co-researcher. Our collaborative, close to practice, practitioner research approach is influenced by the teacher researcher curriculum development approach developed by Lawrence Stenhouse (1975) and informed by the 'inquiry as stance' position of Cochran-Smith and Lytle (2009). The teachers joined the project in response to an open invitation and have all been involved in the Singapore Maths curriculum development project for between one and two years. These are typical of Primary teachers in England who are assigned a class of around 30 children and they teach a broad curriculum to that class, including maths. Six of the teacher researchers have a class in year 1–3 of Primary school (5–8 year olds), one teacher researcher has a year 6 class with children aged 10 and 11. The teachers have between 2 and 25 years of professional experience, six of them are female. The study took place over a two-year period during which the teacher researchers formed part of the research team and were involved to different degrees in literature review, research design, data collection methods and most importantly in collaborative analysis of data. By collaborating with the teachers as practitioner researchers the study aims to contribute to boundary-crossing creation of 'strongly contextualised' and 'socially robust' knowledge (Nowotny, Scott, & Gibbons, 2001).

The study uses an interpretivist methodology to understand teachers' professional learning and development of practice and takes the form of a multiple case study of 7 teachers within 7 Primary schools across the alliance (Yin, 2014). The research data generation focused on one lesson for each teacher researcher and used classroom video including a focus on one pair of children working together within that lesson. The classroom video was captured by a teaching assistant. In addition to classroom video the teacher researcher completed a follow-up interview with the pair of children and captured this as an additional video recording. Semi-structured interviews with each teacher researcher were completed by the research mentor and/or the director of the alliance and these used stimulated recall. This was an attempt to ensure that the teacher's reflections were more grounded in practice (Eraut, 2000; Lyle, 2003). The teacher viewed their classroom video and scanned through using fast forward and pause controls whilst slowing down and commenting on what they felt were distinctive events within the lesson. Towards the end of the interview the teachers were invited to reflect more generally about their practice and the influence of Singapore Maths. The use of classroom video aligned with the aims of our practitioner research approach because of the professional learning benefits of collaborative

analysis that it offers alongside generating rich data. Video has been used effectively in previous studies of curriculum development in mathematics (Lewis, 2014).

This paper reports primarily on our qualitative thematic analysis of the transcriptions of the seven teacher researcher interviews (Braun & Clarke, 2006; Ritchie & Lewis, 2003). The research mentor led the qualitative analysis but worked closely with the school-based director and co-researcher throughout the process. Initial coding of three teacher interview transcripts and a constant comparative approach was used to develop through debate and re-coding an agreed but still evolving index. A collaborative analysis workshop involving teacher researchers coding raw interview transcripts informed the development and checking of the index before it was used to code the remaining transcripts. Continued analysis involved searching for themes that 'capture' important elements within the data and further work focused on refining and confirming the themes and considering the relationships between themes (Nowell, Norris, White, & Moules, 2017). An additional collaborative analysis focus group involved the teacher researchers in pre-reading then debating and helping to shape the emerging analysis. Illustrative quotations are provided in the findings section to make our interpretation as transparent as possible. An additional transcript, included in the analysis, was generated by a teacher researcher focus group questioning the influence of the text books.

In considering the analysis and discussion of findings it is important to note the nature of this small-scale qualitative study including: the cultural and professional location of the study in Primary schools in the North West of England; the small sample of teachers; the fact that these were volunteer teachers with an interest in the mastery maths project; the focus on understanding teacher perspectives; the specificity of the curriculum development project in the subject of maths; and the specificity of the commercially produced scheme with its textbooks and teacher guidance. However, the in-depth nature of the study means that we may seek to 'generalize to theoretical propositions' related to the teacher researchers' classroom practices and their underlying beliefs (Yin, 2014, p. 21).

The ethical risks within this project centre on participating teacher researchers because analysis of classroom video is potentially a risk to professional reputation. Teacher researchers gave formal written consent and have had a right to withdraw at any time. Transcript data was anonymised and pseudonyms are used in publications. Children were told briefly about the research, with adjustment for different age groups, and asked orally for their consent to be in classroom video clips and for a small number of children to complete a short interview. Parents and carers of children involved in the classroom video or short interview video gave informed written consent. The project proposal was scrutinised and approved through the University of Cumbria formal ethical clearance process. Establishing and reflecting on a research ethics framework with teacher researchers is particularly helpful in building trust within the current high accountability context of schools.

8. Findings

The findings are presented in the next two sections reporting on our thematic qualitative analysis. First, a concise summary of changes in teacher strategies is presented, which we characterise as 'framing learning'. These findings are presented more fully in a separate paper focused on whole class exploring of maths problems and providing insight into the nature of the mastery approach lessons (Boyd, & Ash, 2018). Second, a more explicit presentation of the qualitative analysis is presented focused on teacher beliefs in which three themes were identified: struggle and mistakes; mathematical mindsets; and grouping. This presentation uses selected illustrative quotations to provide insight into our coding and interpretation of the data.

8.1. Framing learning within a Singapore maths approach

This concise initial section of the analysis focuses on teacher classroom strategies and the role of the text books. For the purposes of this paper these findings are merely summarised and the underpinning analysis is presented more fully in a separate paper (Boyd, & Ash, 2018). The analysis of lesson video and stimulated recall data suggested that the 7 teacher researchers, at least based on the selected video lessons, were implementing the Singapore Maths mastery approach with fidelity. We characterised the teacher classroom strategy as 'framing' learning and within that overall approach three themes were extracted from the data: dialogue; collaborative exploration; and concept-building. The teachers are facilitating collaborative learning supported by visualisation and questioning to provoke in-depth exploration of the anchor problem and relevant key concepts through verbal reasoning. This strategy includes developing a learning environment that embraces struggle and mistakes and within which the teacher models verbal reasoning to slowly explore problems in depth, rather than demonstrating quick neat calculation to reach the 'correct' answer to questions. The teachers see this move, to the class exploring a problem collaboratively, as a shift from their previous more didactic teaching where the teacher gives a clear explanation of the single correct solution to a maths question followed by guided practice. This exploration phase of the lesson within this type of mastery approach deserves further investigation, especially in relation to teacher questioning and how it provides challenge to students with higher prior attainment.

The teachers foregrounded the role of the textbooks, but at the same time, they emphasise the need to prepare lessons carefully and to engage with the accompanying teacher guidance. The significance of the textbooks and additional teacher guidance became very clear during the initial teacher interview analysis and this led us to hold a teacher focus group specifically engaging with this issue. Each teacher researcher brought a copy of the text book pages for a recent lesson and discussion built from these specific examples to general comments about the significance of the text books:

> It's revolutionised my teaching. My subject knowledge is beyond anything it ever was. I enjoy maths, I have an enthusiasm for maths and I think the depth of rehearsal I go through for my lessons, I would never, ever have had that freedom or time to do it if I didn't have the textbooks (Rachel).

This quotation illustrates the general views of the teachers, that the books were helping them to develop maths content knowledge and understanding. The structure provided by the textbooks seems to free teachers from more mundane aspects of lesson planning. The teachers argued that their lesson preparation is now focused on maths subject knowledge and working out the possible directions in which the children might go in trying to solve the anchor problem. For example, one anchor problem involves comparing how far two children have run (1 km 20 m and 1.2 km). In planning for this lesson the teacher will realise how understanding decimals and their relationship with fractions leads to the main lesson focus of converting units of measure. The teacher learning during lesson planning seems to be provoked by trying to understand the careful choice of task by the text book author:

> I think if I'd just picked this textbook off the shelf and delivered the lesson, I wouldn't have got half as much out of it though,

> because it was just by sitting and looking at it at home, I thought, 'that's interesting why have they done that?' It's just the way it's presented here that's so interesting because children will write things down in columns and I like the way it's got that.
>
> (teacher during text book focus group)

In this way it seemed that the exploring phase of the lessons, despite the central role of the text books, appears to demand development of subject knowledge by teachers.

8.2. Struggle, mathematical mindsets and grouping

This section of the analysis focuses on evidence of teacher beliefs. The teachers were fully involved in implementing the Maths - No Problem!™ scheme in their maths lessons and this analysis highlights teachers' beliefs as they implement the new classroom strategies. Three themes are presented and illustrated with selected quotations: struggle and mistakes; mathematical mindsets; and grouping.

8.2.1. Struggle and mistakes

All of the teachers raised the issue of struggle and mistakes and adopted a positive view of the learning potential:

> … I think if they can actually identify that they've made a mistake themselves, that they can fix it themselves. For me personally, it's about a mind-set because it means, 'oh if I make a mistake it's OK, because I can just check it and I can actually do it right next time'. I don't want them to go to [a teacher]: am I right; am I wrong? I want them to have the confidence to check themselves: are they right; are they wrong? Or to use their partner beside them as well … that's why it's 'can you check your partners to see their counting' and they're checking each other (Veronica).

The teacher claims here to be more patient and willing to wait for the children to solve problems themselves without intervention. This quotation also illustrates the shift towards collaborative learning. In some instances, the teacher revealed a diplomatic way of handling a mistake by a child by allowing peers to question them rather than directly intervening as a teacher:

> I didn't recognise the misconception; one of the other children said that that was wrong. So I didn't say she was right or wrong and I still haven't said that she's right or wrong but she's now corrected herself and the rest of the class said 'it's three twelfths' so I said 'can we agree it's three twelfths?' So that must be right then if they all agreed. So I've not actually told her that she was wrong … (Kirsty).

In this case Kirsty is allowing the children to collectively work on the problem rather than zoom in on the mistake.

In some ways the attitude to struggle and mistakes goes beyond the children's activity and attitudes and influences the next layer up of the teacher's practice. This allows the teacher to model being a learner at the level of Maths problems but also at the level of teaching Maths:

> I'm far happier to be the person making mistakes at the front or not getting things right and I'm less frightened about mistakes in the lesson. It doesn't worry me now if things aren't going the right way. They're not going the right way and we use that within the lesson. If the lesson isn't going the right way … as long as you're including the children within that because they're part of it, you've got that time to say 'this isn't going the right way. Talk to each other. Why isn't this going in the right way? Why can't we get this? What's not right here?' (Rachel).

In this case the teacher seems to be revealing a more collaborative endeavour within the classroom with a more equal power distribution and a more explicit role for the children in shaping the approach to teaching and learning.

8.2.2. Mathematical mindsets

The engagement with Singapore Maths may be related to the teachers' confidence and attitude to learning Maths and being a mathematician:

> Yeah. It is a mind-set thing so instead of focusing onto an answer and judging things by speed a lot of the time and how quickly can I–I know that I've got strategies; I can come to the right answer or I can find out mathematical ways of thinking and learning and the more that I do that, so the more I've taught these lessons, the more those things become clearer … (Andrew).

The teacher reflects the significant shift away from 'School Maths' as an activity that requires speed and swift resolution to the 'correct' answer. However, this quotation also suggests that the teachers may attribute their own developing confidence in becoming a mathematician, with its associated benefits for the children, to their engagement with Singapore Maths.

In the interviews the teachers seem to be trying to resolve their beliefs about mathematical 'ability' with their engagement with the Singapore Maths approach:

> I think with maths you're continually learning. You're learning different ways; you're learning different methods. I know when I was at school it was all about conventions … whereas now, as a teacher, I'm learning new methods … so I think you're always learning and your intelligence is not capped. You've just got to be open-minded; you've got to be open to new learning; new methods; new ways of understanding maths and it's a case of you are always learning, you're increasing the amount of intelligence you have in maths … (Kirsty).

This comment reveals how Kirsty understands growth mindset in relation to Singapore Maths and the layered way in which she is referring to her own beliefs and learning as well as that of her children is a common aspect within the teachers' talk. All of the teachers are proud of the level of children's engagement during their Singapore Maths lessons and they relate this to children of all levels of prior attainment and usually refer to this as 'ability':

> Like if you looked around that classroom now, even in the video, you wouldn't think 'oh they're your best mathematicians and they can't do maths'. I don't think you could tell in the video or even if you were in the classroom, unless you went and talked to the children, I don't think you would be able to tell as an observer who was struggling. They're all doing something. You'd probably be able to spot the 'more able' from what they say … (Patricia).

In referring to 'mindset' the teachers are not using mindset theory in a tightly defined way and ascribing a particular fixed or growth mindset to individual children, although they are likely to have come across the theory to different degrees. Rather, they are using mindset within the specific context of Maths and this is

entangled with beliefs about what being a mathematician involves and with the complexity of classroom practice:

> … it's that idea that you are building a maturity as a learner … so it is that … growth mindset, that resilience … and also that mistakes actually become a central process. So, almost, now in our lessons we'll be quite glad when somebody makes a mistake because, you know, it's great, something to really, you know … something to run with … and you're also modelling that, you know those skills of self-checking … so the child's talking out loud about what they're doing and they might make a mistake, they're self-checking and self-regulating which is actually a really important skill (Rachel).

This quotation illustrates the way that developing a classroom learning environment that embraces struggle and mistakes is intertwined with collaborative learning, formative assessment and development through metacognition as a self-regulated learner. This illustrative teacher quotation provides insight into teachers' expertise and the limitations of intervention studies that try to measure the impact of such elements of practice in isolation.

In association with high levels of engagement the teachers claim that the mastery approach to Maths helps to avoid labelling children by prior attainment, although in the discussion and in their planning the teachers continue to refer to children using 'ability' labels:

> Having an anchor task that's open to all, I think it's really helped the children who are 'lower ability' because it's their self-esteem … the 'lower ability' child who may be using the equipment can still represent the equipment on a whiteboard with drawings and we'll have that up as one of the methods as well and they don't see their method as being different to anybody else's … they still feel that they are part of the lesson … their self-esteem has grown; their understanding … and they're now seeing themselves as a mathematician … (Kirsty).

In this quotation the teacher acknowledges that having a low threshold task is only part of the social process of building a positive learning environment and that the handling of children's suggested solutions to the problem at different levels of sophistication is also critical. The quotation represents the very positive experience of the teachers in relation to the Singapore Maths approach, especially in relation to engagement by children but also to the way that children are coming to see themselves as competent mathematicians.

8.2.3. Grouping

Partly influenced by the self-esteem issue, the teachers have slightly varied ways of organising children within their classrooms. Andrew uses random selection of 'talking partners' and changes them regularly. Rachel, Kirsty, Lucy and Patricia all put the children in pairs selected deliberately to have different prior attainment in maths. Audrey and Veronica use the same approach but pair the children based on 'how well they cooperate together' and what type of 'temperament' they have as learners in maths. Rachel, Veronica and Patricia also use 'zoning', so that there are clusters of pupils in particular areas of the classroom, for example around one group table, that include pairs where one child has special learning needs or is considered to have a particularly high level of prior attainment. Overall, the teachers' beliefs about in-class grouping appear to be consistent with the idea of a growth mindset, as opposed to focusing on fixed ability levels:

> So they're all sat 'mixed ability'. The children who - the most fragile learners, are usually sat with a 'high ability' child. Not necessarily a 'high ability' child but somebody who will be able to explain their thinking to the child who's struggling. And I do think that's a big shift as well from the way I would have taught maths before and did this class before, it would be very much separated into, you know, 'you can only do this because you're 'low ability, middle and high' whereas now I feel like the 'high or middle ability' children can help the other children by showing them the equipment and talking to them about it and also I think the reasoning and the talking from the 'high or middle ability' children, helps them as well to distil what they're thinking by explaining it to somebody else (Lucy).

All of the teachers feel that the Singapore Maths approach creates less need for grouping by prior attainment or 'ability':

> … it is more about that facilitating learning. You're presenting the children with an opportunity, they come out with the outcomes themselves; the reasoning themselves. There's even less of a need for the children to be 'ability' grouped … (Andrew).

This rationale for moving away from in-class grouping by prior attainment, partly through a shift towards developing classroom dialogue, is claimed by four of the teachers to be affecting the approach in other curriculum subjects within their school. It is difficult to disentangle the impact of the Singapore Maths scheme within a general shift towards abandoning grouping by prior attainment that may be influenced by other factors in each school. However, in relation to both mindset and mixed grouping the teachers are finding that the Singapore Maths approach seems at least to be demonstrating some benefits and possibilities for developing growth mindsets, for effective whole class teaching placing children in mixed pairs and for abandoning in-class grouping by prior attainment.

The beliefs of teachers around grouping are implicated in the wider pernicious influence of the high stakes inspection system in England, referred to as 'Ofsted'. Teachers and especially their head teachers are continually second-guessing what the inspectors will be looking for during inspections and especially during classroom observations. Ofsted are an influential force running alongside the overall pressure for high test results:

> … this is where you've got that terrible tension between what you as a professional want, what your core value is in terms of teaching maths for the children, and then you've got your accountability-driven system, that's where that tension is. Often if you're talking to Year 6 teachers, 'well I've got to get them to the SATS [national tests], I can't afford the time to do this', and yet you absolutely know this approach works … I think that's where we're in a real tension place … it does give you permission but the wider picture doesn't allow for that permission perhaps …
>
> (teacher researcher during collaborative analysis).

Despite this tension, the scheme gives teachers 'permission' to spend time in Maths exploring and helps them to resist focusing on coverage and teaching to the test. This is where day to day classroom practice becomes cultural change and the nature of the mastery lessons as well as the process of implementing the mastery approach have perhaps both contributed:

> … I think there's very much a culture of acceptance that starts to be built through school that the teachers are learners too

> because … I think the whole of the process has made the children think about themselves as a learner, their teachers as a learner in a way that they would never ever have done before in a traditional maths lesson.
>
> (teacher researcher during collaborative analysis).

This quotation illustrates the layered nature of learning in Maths that the teachers all highlight, and it has resonance with Hattie's concept of 'visible learning' (Hattie, 2012) with children and teacher seeing through each other's eyes. This study has revealed that teachers and their students show signs of identifying as mathematicians, which may be related to their engagement with the mastery maths approach.

9. Discussion: strategies and beliefs

Planning for mastery approaches to maths, in this case for the new teacher strategy of exploring an anchor problem, appears to involve a change in lesson preparation so that it focuses more on maths subject knowledge. This subject knowledge preparation is necessary because the children might take the anchor problem, provided by the textbook and introduced by the teacher, and go in different directions. Thus teacher planning, to enable their successful framing of collaborative learning through exploration, relies heavily on the textbooks. However, the lessons provided by the textbook embed the pedagogical approach and in particular give the teacher 'permission' to spend classroom time on collaborative exploration of the anchor problem. This permission helps to counteract contextual pressures to rush on with curriculum delivery from an overloaded national curriculum, high stakes external inspection, and the emphasis on test results and school league tables. The dependence on the textbooks and associated teacher guidance creates a worry, because it might be part of reducing teachers to a technician role of 'delivering' the curriculum. It also creates a considerable pressure to be confident that the textbooks and guidance being followed are evidence-based and effective (Oates, 2014). There is also a considerable investment involved for schools in committing to a commercially produced mastery maths scheme. However, the focus on exploring and on dialogue appears to keep the skill and subject knowledge of the teacher firmly at the heart of this mastery approach to maths. Therefore, the books in themselves are insufficient and only provide one element of the approach.

Grouping, or moving away from in-class grouping based on prior attainment, and the associated belief in differentiation by task, is an important shift identified in this study. In England there is a very well-established dependence on in-class grouping by prior attainment in maths in Primary schools (and on setting in secondary schools). The commitment to grouping and setting in schools in England has proved remarkably persistent. This is despite the research evidence that such grouping or setting is of only limited benefit to children with higher levels of prior attainment and certainly damages the progress of children in lower groups or sets (Francis et al., 2017). The analysis indicates that teachers' beliefs were consistent with the Singapore Maths approach and further research should investigate the influence of engagement with the text-book scheme on teachers' beliefs. The teachers do not see any value for in-class grouping within the mastery Maths approach because it does not require differentiation by task, the children are all working on the same materials during the dialogic exploring phase of the lesson. There are signs that this change in beliefs around the effectiveness of in-class grouping is influencing practice across the curriculum rather than only in maths lessons. Despite these signs of change in strategies and beliefs, the teacher researchers persist in their habit of describing children as high or low 'ability'. Perhaps the high accountability context in which they work helps to explain this persistence in labelling children. At the least, this suggests that during implementation of mastery maths curriculum development projects, there is a need to explicitly engage with conceptions of intelligence and mindset.

Intelligence in maths may be conceived as fixed or malleable. Many teachers in England have come across this idea through varied levels of engagement with the mindset theory of Dweck (1999; 2006). Early research on mindset theory tended to focus on general mindset or on mindset broadly related to 'school work'. It is interesting to focus on mindset in a more specific context, such as within school mathematics. Our analysis suggests that engagement in the mastery maths intervention may be associated with changing teacher beliefs related to becoming a mathematician, including adopting a more malleable conception of intelligence in the context of maths, however, this remains speculative. Teachers are reflecting both on their own mindset within the school subject of maths and on the 'situated' domain specific mindset of their students. This reflection on beliefs about intelligence is entangled with a changing understanding of the nature of mathematics and of being a mathematician. The analysis shows how the teacher researchers are seeing the subject of maths as being about collaborative problem-solving and deep thinking, rather than focusing on speed and calculation to reach a single ideal solution. It is as much a shift in cultural beliefs about the subject of maths and of 'ability' within maths as it is about changing beliefs about the malleable nature of intelligence.

Steps in recontextualisation (Bernstein, 2000) of a subject discipline so that, for example, mathematics becomes 'school maths', may include writing national curriculum documents, creating national tests, writing text books and the influence of school inspection systems. These all form part of what Bernstein referred to as the ORF, the 'official recontextualising field'. In contrast, the PRF, or 'pedagogic recontextualising field' consists of the teachers in schools and includes teacher educators based in education departments. 'If the PRF can have an effect on pedagogic discourse independently of the ORF, then there is some autonomy and struggle over pedagogic discourse and its practices' (Bernstein, 2000, p. 33). The ORF has become dominant in England and has imposed a performance mode, particularly since the Thatcher government of the 1980s and continuing through successive governments. The shift towards mastery maths, including adapting South Asian approaches, has been introduced by the ORF, for example through changes to the national curriculum and in response to political anxiety about the position of England in international comparisons of maths performance, but ironically this shift is coming up against teacher beliefs within the PRF. Addressing cultural values and beliefs of teachers and students is an essential element of curriculum development because teaching is relational. Our analysis suggests that beliefs about intelligence, though culturally held and developed widely throughout society, are also shaped within classroom relationships and the perceived nature of curriculum subject disciplines. The concept of growth mindset is contested, not least because it was developed as if it were a generic or decontextualized belief (Dweck, 1999). This study has pursued mindset as a domain specific belief, as 'mathematical mindset' (Boaler, 2016) and suggests that such cultural beliefs related to curriculum subject disciplines are an important element that may be understood as part of Bernstein's rules of recontextualisation.

10. Conclusion

The analysis provides insight into the teacher researchers' changing beliefs about the nature of school maths, the place of struggle and mistakes, expectations, mindset and grouping. It is possible that in some cultures maths particularly lends itself to changing teacher expectations (Rubie-Davies, Peterson, Sibley, & Rosenthal, 2015) not least because 'school maths' has become such a strange beast that is so far from the subject discipline of 'mathematics' (Hudson, Henderson, & Hudson, 2015; Bernstein, 2003). However, in the ongoing struggle to shape the curriculum the examiners, the designers of national tests for Primary pupils, currently hold an influential position because they control the evaluative rules of school maths (Bernstein, 2000; Puttick, 2015). The Singapore Maths scheme appears to have sufficient traction and priority for the teachers in this study so that it is changing the rules by which mathematics is transformed to become school maths. The teachers seem to be able to resist the pressure around test results and focus more on engagement and concept-building. The emphasis on verbal reasoning and exploring within the Singapore Maths approach perhaps enhances teacher agency and autonomy within the classroom. This is associated with signs that they may be changing their underpinning beliefs around mindset and this has implications for their practice and understanding of grouping by prior attainment. Without reaching substantial conclusions, nonetheless the study has usefully considered the contested concept of growth mindset as domain specific, as situated mathematical mindset: a belief held in varied ways by teachers and children, that the more you practice at the edge of your current attainment level in maths, the more intelligent you will become as a mathematician (Boaler, 2016).

The weak influence on policy and practice in England of the research evidence on grouping by prior attainment is a puzzle. Francis et al. identify a discourse of the 'natural order' by which aspirational parents provide political support for grouping by prior attainment because they believe their children will populate top sets (2016). These researchers are aiming to produce influential randomized control trial evidence that might influence the debate by adding persuasive evidence to the research evidence. The Singapore Maths mastery approach is an example in which teachers' beliefs around mindset and grouping are being challenged and changed in their classrooms and embedded in the teaching of curriculum subjects as an element of Bernstein's pedagogic recontextualising field (2003). In this sense, such interventions supported by teacher inquiry, embedded in subjects such as school maths, offer a bottom-up approach to research-informed curriculum development.

The findings of this small-scale study are important because of the implications for teachers' professional learning. We would speculate that implementation of a well-designed textbook scheme, one that supports a pedagogical approach informed by theory and research, has the potential to do more than expand teachers' repertoire of classroom strategies, it may also provoke change in embedded teacher beliefs that have proved resilient in previous reform efforts. Teachers and other school leaders in England are wise to be wary of text book schemes that may reduce teacher agency to the level of technician, but this study suggests that text book schemes are worth careful and critical consideration.

Funding

This research project was funded by the Deep Learning Teaching Schools Alliance.

Acknowledgements

Seven teacher researchers contributed significantly to this research project, they are Lucy Evans, Ann Kirk, Rosie Ross, Paula Spenceley, Vicky Stout, Adam Vasco and Keri Williams.

References

Ball, S. (2013). *The education debate* (2nd). Bristol: Policy Press.
Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*. Upper Saddle River NJ: Prentice-Hall.
Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: W. H. Freeman and Company.
Bernstein. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique* (Rev). Oxford: Rowman & Littlefield.
Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127–147. https://doi.org/10.1007/s10649-011-9333-2.
Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development, 78*(10), 246–263. https://doi.org/10.1111/j.1467-8624.2007.00995.x.
Block, J. H., & Anderson, L. W. (1975). *Mastery learning in classroom instruction*. New York: Macmillan.
Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco: Jossey-Bass.
Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experiences of ability grouping – disaffection, polarisation and the construction of failure. *British Educational Research Journal, 26*(5), 631–648. https://doi.org/10.1080/01411920020007832.
Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal, 34*(2), 167–194.
Boyd, P., & Ash, A. (2018). Teachers framing exploratory learning within a text-book based Singapore Maths mastery approach. *Teacher Educator Advancement Network Journal, 10*(1), 62–73. Retrieved from: https://ojs.cumbria.ac.uk/index.php/TEAN/article/view/442.
Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*, 77–101. https://doi.org/10.1191/1478088706qp063oa.
Brown, T., & McNamara, O. (2011). *Becoming a mathematics teacher*. London: Springer.
Bruner, J. (1966). *Toward a theory of instruction*. Harvard University Press.
Claro, S., Paunesku, D., & Dweck, C. (2016). Growth mindset tempers the effects of poverty on achievement. *Proceedings of the National Association of Sciences of the USA, 113*(31), 8664–8668. Retrieved from: http://www.pnas.org/content/113/31/8664.full.
Cochran-Smith, M., & Lytle, S. L. (2009). *Inquiry as stance: Practitioner research for the next generation*. New York: Teachers College Press.
Cockroft, W. H. (1982). *Mathematics counts*. London: HMSO.
Cohen, E. G., & Lotan, R. A. (2014). *Designing groupwork: Strategies for the heterogeneous classroom* (3rd Ed.). New York: Teachers' College Press.
Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16*(5), 324–336. https://doi.org/10.2307/749355.
Correa, C. A., Perry, M., Sims, L. M., Miller, K. F., & Fang, G. (2008). Connected and culturally embedded beliefs: Chinese and US teachers talk about how their students best learn mathematics. *Teaching and Teacher Education, 24*(1), 140–153. https://doi.org/10.1016/j.tate.2006.11.004.
Cross, D. I. (2009). Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education, 12*, 325–346. https://doi.org/10.1007/s10857-009-9120-5.
Dweck, C. (1999). *Self-theories: Their role in motivation, personality and development*. New York: Psychology Press.
Dweck, C. S. (2006). *Mindset: The new psychology of success*. New York: Random House.
EEF [Education Endowment Fund]. (2015). Teaching and learning toolkit: An accessible summary of educational research on teaching 5-16 year olds. *Education Endowment Fund & Sutton Trust*. Retrieved from: https://educationendowmentfoundation.org.uk/evidence/teaching-learning-toolkit.
Eraut, M. (2000). Non-formal learning and tacit knowledge in professional work. *British Journal of Educational Psychology, 70*(1), 113–136.
Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403–434. https://doi.org/10.2307/749875.
Francis, B., Archer, L., Hodgen, J., Pepper, D., Taylor, B., & Travers, M. C. (2017). Exploring the relative lack of impact of research on 'ability grouping' in England: A discourse analytic account. *Cambridge Journal of Education, 47*(1), 1–17. https://doi.org/10.1080/0305764X.2015.1093095.
Gabriele, A. J., & Joram, E. (2007). Teachers' reflections on their reform-based teaching in mathematics: Implications for the development of teacher self-

efficacy. *Action in Teacher Education, 29*(3), 60–74.
Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. New York: Routledge.
Heine, S. J., Kitayama, S., Lehman, D. R., Takata, T., Ide, E., Leung, C., et al. (2001). Divergent consequences of success and failure in Japan and North America: An investigation of self-improving motivations and malleable selves. *Journal of Personality and Social Psychology, 81*(4), 599–615. https://doi.org/10.1037/0022-3514.81.4.599.
Hudson, B., Henderson, S., & Hudson, A. (2015). Developing mathematical thinking in the primary classroom: Liberating students and teachers as learners of mathematics. *Journal of Curriculum Studies, 47*(3), 374–398. https://doi.org/10.1080/00220272.2014.979233.
Hymer, B., & Gershon, M. (2014). *The growth mindset pocketbook*. Alresford, UK: Teachers' Pocketbooks.
Jerrim, J. (2015). Why do East Asian children perform so well in PISA? An investigation of Western-born children of East Asian descent. *Oxford Review of Education, 41*(3), 310–333. https://doi.org/10.1080/03054985.2015.1028525.
Jerrim, J., & Vignoles, A. (2016). The link between East Asian 'mastery' teaching methods and English children's mathematics skills. *Economics of Education Review, 50*, 29–44. https://doi.org/10.1016/j.econedurev.2015.11.003.
Klassen, R. M. (2004). A cross-cultural investigation of the efficacy beliefs of South Asian immigrant and Anglo Canadian non-immigrant early adolescents. *Journal of Educational Psychology, 96*(4), 731–742. https://doi.org/10.1037/0022-0663.96.4.731.
Kutnick, P., Sebba, J., Blatchford, P., Galton, M., & Thorp, J. (2005). *The effects of pupil grouping: Literature review*. London: Department for Education and Skills.
Lakatos, I. (1976). *Proofs and refutations*. Cambridge: Cambridge University Press.
Lazarides, R., Buchholz, J., & Rubach, C. (2018). Teacher enthusiasm and self-efficacy, student-perceived mastery goal orientation and student motivation in mathematics classrooms. *Teaching and Teacher Education, 69*(1), 1–10. https://doi.org/10.1016/j.tate.2017.08.017.
Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*(1), 91–102. https://doi.org/10.1007/s10857-006-9006-8.
Lewis, G. M. (2014). Implementing a reform-oriented pedagogy: Challenges for novice secondary mathematics teachers. *Mathematics Education Research Journal, 26*(2), 399–419. https://doi.org/10.1007/s13394-013-0092-5.
Lyle, J. (2003). Stimulated recall: A report on its use in naturalistic research. *British Educational Research Journal, 29*(6), 861–878. https://doi.org/10.1080/0141192032000137349.
Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences, 16*, 404–406.
Mazzocco, M. M. M., Hanich, L. B., & Noeder, M. M. (2012). Primary school age students' spontaneous comments about math reveal emerging dispositions linked to later mathematics achievement. *Child Development Research, 2012*, 170310. https://doi.org/10.1155/2012/170310. Retrieved from https://www.hindawi.com/journals/cdr/2012/170310/ .
Meirink, J. A., Verloop, N., & Bergen, T. C. M. (2009). Understanding teacher learning in secondary education: The relations of teacher activities to changed beliefs and teaching and learning. *Teaching and Teacher Education, 25*, 89–100. https://doi.org/10.1016/j.tate.2008.07.003.
Michaelides, M. (2008). Emerging themes from early research on self-efficacy beliefs in school mathematics. *Electronic Journal of Research in Educational Psychology, 6*(1), 219–234.
Micklewright, J., Jerrim, J., Vignoles, A., Jenkins, A., Allen, R., Ilie, S., et al.Hein, C. (2014). *Teachers in England's secondary schools: Evidence from TALIS 2013*. Department for Education Research Report DFE-RR302. Retrieved from https://www.gov.uk/government/publications/teachers-in-secondary-schools-evidence-from-talis-2013.
Moscardini, L. (2014). Developing equitable elementary mathematics classrooms through teachers learning about children's mathematical thinking: Cognitively Guided Instruction as an inclusive pedagogy. *Teaching and Teacher Education, 43*, 69–79. https://doi.org/10.1016/j.tate.2014.06.003.
NCETM (National Centre for Excellence in the Teaching of Mathematics). (2016). The essence of maths teaching for mastery. Retrieved from https://www.ncetm.org.uk/files/37086535/The+Essence+of+Maths+Teaching+for+Mastery+june+2016.pdf .
Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods, 16*, 1–13. https://doi.org/10.1177/1609406917733847.
Nowotny, H., Scott, P., & Gibbons, M. (2001). *Rethinking Science: Knowledge and the public in an age of uncertainty*. Cambridge: Polity Press.
Oates, T. (2014). *Why textbooks count: A policy paper*. Cambridge: University of Cambridge.
Pajares, F., & Kranzler, J. (1995). Self-efficacy beliefs and general mental ability in mathematical problem-solving. *Contemporary Educational Psychology, 20*, 426–443. https://doi.org/10.1006/ceps.1995.1029.
Paolucci, C. (2015). Changing perspectives: Examining the potential for advanced mathematical studies to influence pre-service teachers' beliefs about mathematics. *Teaching and Teacher Education, 49*, 97–107. https://doi.org/10.1016/j.tate.2015.03.002.
Polya, G. (1957). *How to solve it* (2nd ed.). London: Penguin Books.
Priestley, M., Biesta, G., & Robinson, S. (2015). *Teacher agency: An ecological approach*. London: Bloomsbury.
Puttick, S. (2015). Chief examiners as prophet and priest: Relations between examination boards and school subjects, and possible implications for knowledge. *Curriculum Journal, 26*(3), 468–487. https://doi.org/10.1080/09585176.2014.1000936.
Ramirez, G., Chang, H., Maloney, E. A., Levine, S. C., & Beilock, S. L. (2016). On the relationship between math anxiety and math achievement in early elementary school: The role of problem solving strategies. *Journal of Experimental Child Psychology, 141*, 83–100. https://doi.org/10.1016/j.jecp.2015.07.014.
Ritchie, J., & Lewis, J. (2003). *Qualitative research practice: A guide for social science students and researchers*. London: Sage.
Rogers, M. A. P., Cross, D. I., Gresalfi, M. S., Trauth-Nare, A. E., & Buck, G. A. (2011). First year implementation of a project-based learning approach: The need for addressing teachers' orientations in the era of reform. *International Journal of Science and Mathematics Education, 9*(4), 893–917. https://doi.org/10.1007/s10763-010-9248-x.
Rubie-Davies, C. M., Peterson, E. R., Sibley, C. G., & Rosenthal, R. (2015). A teacher expectation intervention: Modelling the practices of high expectation teachers. *Contemporary Educational Psychology, 40*, 72–85. https://doi.org/10.1016/j.cedpsych.2014.03.003.
Schunk, D. H. (1991). Self-efficacy and academic motivation. *Educational Psychologist, 26*(3–4), 207–231.
Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*(4), 387–402.
Stenhouse, L. (1975). *An introduction to curriculum research and development*. London: Heinemann.
Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213–226. https://doi.org/10.1016/S0742-051X(00)00052-4.
Swann, M., Peacock, A., Hart, S., & Drummond, M. J. (2012). *Creating learning without limits*. Maidenhead, UK: Open University Press.
Tschannen-Moran, M., & Woolfolk Hoy, A. (2001). Teacher efficacy: Capturing an elusive construct. *Teaching and Teacher Education, 17*, 783–805. https://doi.org/10.1016/S0742-051X(01)00036-1.
Wong, K. Y., Lee, P. Y., Kaur, B., Foong, P. Y., & Ng, S. F. (2009). *Mathematics education: The Singapore journey*. Singapore: World Scientific Publishing.
Yew Hoong, L., Weng Kin, H., & Lu Pien, C. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *Mathematics Educator, 16*(1), 1–19. Retrieved from: http://math.nie.edu.sg/ame/matheduc/tme/tmeV16_1/TME16_1.pdf.
Yin, R. K. (2014). *Case study research: Design and methods* (5th ed.). London: Sage.